



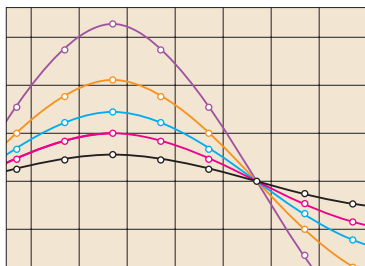
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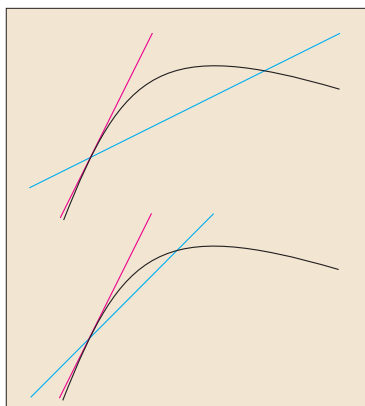


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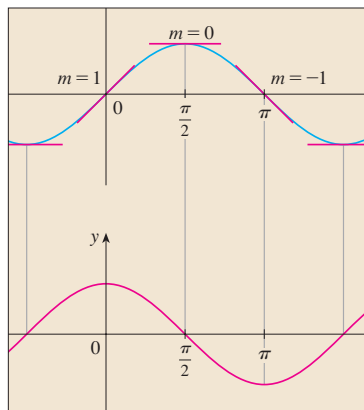
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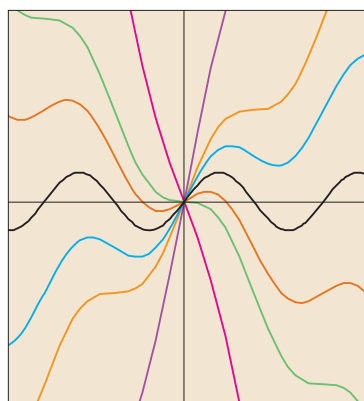
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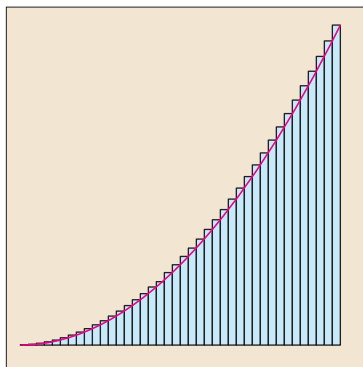
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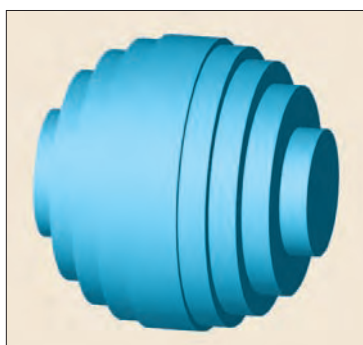
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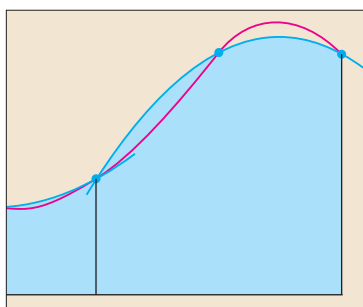
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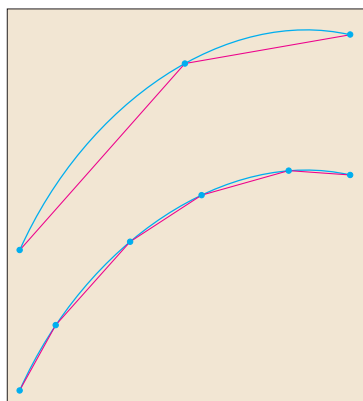


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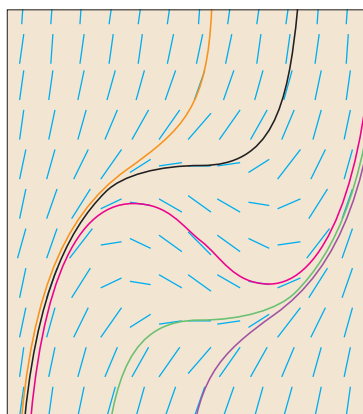
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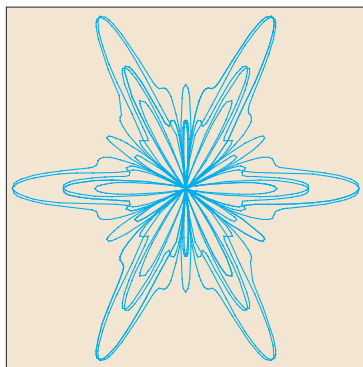
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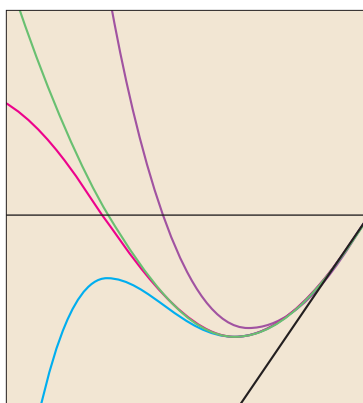
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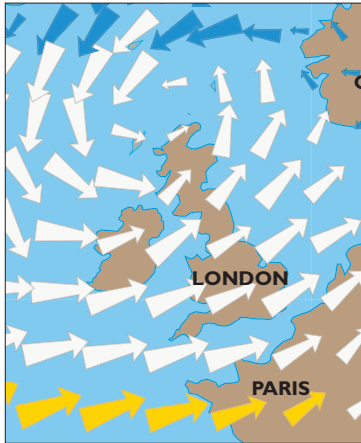
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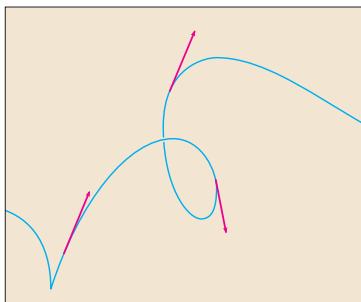
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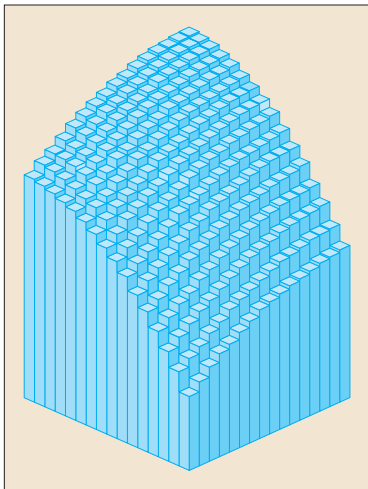


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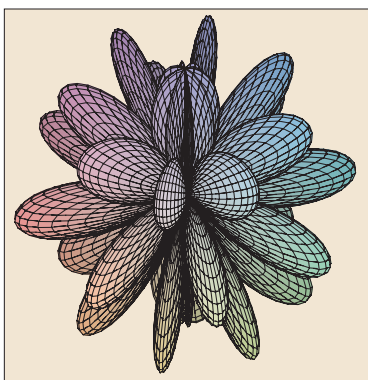
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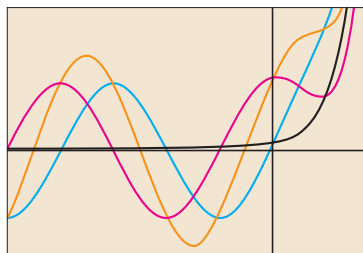
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PREFACE

A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.

GEORGE POLYA

The art of teaching, Mark Van Doren said, is the art of assisting discovery. I have tried to write a book that assists students in discovering calculus—both for its practical power and its surprising beauty. In this edition, as in the first five editions, I aim to convey to the student a sense of the utility of calculus and develop technical competence, but I also strive to give some appreciation for the intrinsic beauty of the subject. Newton undoubtedly experienced a sense of triumph when he made his great discoveries. I want students to share some of that excitement.

The emphasis is on understanding concepts. I think that nearly everybody agrees that this should be the primary goal of calculus instruction. In fact, the impetus for the current calculus reform movement came from the Tulane Conference in 1986, which formulated as their first recommendation:

Focus on conceptual understanding.

I have tried to implement this goal through the *Rule of Three*: “Topics should be presented geometrically, numerically, and algebraically.” Visualization, numerical and graphical experimentation, and other approaches have changed how we teach conceptual reasoning in fundamental ways. More recently, the Rule of Three has been expanded to become the *Rule of Four* by emphasizing the verbal, or descriptive, point of view as well.

In writing the sixth edition my premise has been that it is possible to achieve conceptual understanding and still retain the best traditions of traditional calculus. The book contains elements of reform, but within the context of a traditional curriculum.



ALTERNATIVE VERSIONS

I have written several other calculus textbooks that might be preferable for some instructors. Most of them also come in single variable and multivariable versions.

- *Calculus*, Sixth Edition, is similar to the present textbook except that the exponential, logarithmic, and inverse trigonometric functions are covered in the second semester.
- *Essential Calculus* is a much briefer book (800 pages), though it contains almost all of the topics in *Calculus*, Sixth Edition. The relative brevity is achieved through briefer exposition of some topics and putting some features on the website.
- *Essential Calculus: Early Transcendentals* resembles *Essential Calculus*, but the exponential, logarithmic, and inverse trigonometric functions are covered in Chapter 3.